

NON-HERMITIAN JACOBI-NATANZON SYSTEMS, $SO(4, \mathbb{C})$ LOOP MULTIPLIERS, AND FINITE-QUOTIENT SQUEEZING

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Abstract

We construct a PT-symmetric complex extension of a Jacobi-Natanzon potential and study a real-energy bound-state sector through a Liouville reduction to the Jacobi equation. By considering a complexified Jacobi-Natanzon family, of which the PT-symmetric system is a special case, and using the complex-parameter continuation of the algebraic-Jacobi-function ladder construction within the Jacobi-polynomial sector, we develop the potential-algebra description in terms of two commuting complex ladder structures realizing the Lie algebra $\mathfrak{sl}(2, \mathbb{C}) \oplus \mathfrak{sl}(2, \mathbb{C}) \simeq \mathfrak{so}(4, \mathbb{C})$. The ladder action admits an integer-shift interpretation on the space of potential labels, where closed-loop products of Jacobi-ladder maps can acquire nontrivial complex multipliers, with corresponding real finite-dimensional loops being phase-trivial. We further introduce algebraic squeezed states on finite irreducible quotients of the complex highest-weight ladder modules and describe their quadrature fluctuations with respect to an auxiliary algebraic inner product. Their basis classes carry labels drawn from finite collections of Jacobi-Natanzon Hamiltonians.