LINEAR HEAT EQUATION IN PERIODIC DOMAINS

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Abstract

We treat the classical linear heat equation in unbounded, periodic waveguides $\Pi \subset \mathbb{R}^d$ by using the Floquet-Bloch transform methods. As well known, the behavior of the solution of the heat equation is determined by the underlying spectral Laplace problem. The Floquet-Bloch transform F turns the elliptic problem on the unbounded domain Π into a corresponding elliptic model problem on the bounded periodic cell ϖ of Π. The Floquet-Bloch transform can also be directly applied to the original heat equation, which yields a heat equation with mixed boundary conditions on the periodic cell ϖ. We analyse the connection between the two approaches to the problems. In the case of a disjoint component of the essential spectrum, we describe the corresponding spectral projection and note that the translated Wannier functions form an orthonormal basis in $\mathcal{H}_S$. Applications to the heat equation follow immediately.

In the case Π is periodic in d directions, we observe that for a general, integrable initial data, the solution decays for large t at the same rate $t^{-d/2}$ as in the case of the Cauchy problem in the entire Euclidean space, but initial data with certain vanishing conditions for the x-integral leads to faster decay rates: given any positive integer N, we find sufficient conditions for the initial data so that the solution decays at least at the rate t^{-N} .

References

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