QUANTUM GRAPHS WITH COULOMB-TYPE POTENTIALS

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Abstract

In recent decades, the theory of differential operators on metric graphs has been the subject of extensive and systematic study, particularly due to its numerous applications in solid-state physics and engineering. From a physics standpoint, however, the most exciting application of this theory is quantum graphs. Quantum graphs provide effective mathematical models that allow us to study quantum systems in a framework that makes explicit solutions possible.

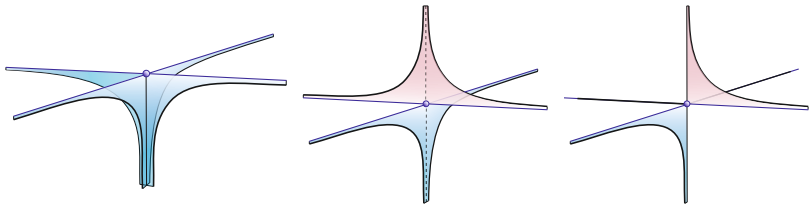

Figure 1: The "classic" and exotic Coulomb potentials.

In this talk, we will construct exactly solvable models of quantum graphs with Coulomb-type potentials whose singularities are located at the vertices (see Fig. 1). This problem generalizes the well-known problem of constructing a one-dimensional model of a hydrogen atom [1]. We describe all self-adjoint realizations of formal Coulomb Hamiltonians on a star graph. Regularizing these potentials answers the question of which vertex interactions are physically motivated for the Coulomb potentials. We establish conditions under which Schrödinger operators with cut-off Coulomb potentials coupled with $(\alpha\delta' + \beta\delta)$-like ones converge in the norm resolvent topology. The 1D Coulomb potentials and the δ'-pseudopotential are very sensitive to their regularization method. In particular, the one-dimensional model of the hydrogen atom shows a critical dependence on the behaviour of cut-off Coulomb potentials at small

distances [2]. These properties are also inherent in quantum graphs with Coulomb potentials.

This talk is based on work [3].

References

[1] Loudon, R. One-dimensional hydrogen atom. Proc. R. Soc. A Math. Phys. Eng. Sci. 472(2185), 20150534 (2016).

[2] Golovaty, Y. 1D Schrödinger operators with Coulomb-like potentials. J. Math. Phys. 60(8), 082105 (2019).

[3] Golovaty, Y. Quantum graphs: Coulomb-type potentials and exactly solvable models. Ann. Henri Poincaré 24, 2557-2585 (2023).