

NEAR-ZERO EIGENVALUES AND RESONANCES FOR SIGN-INDEFINITE TRANSMISSION CONDITIONS

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Abstract

I consider an analytic family of operators of the form

$$L(\varepsilon) := -(\chi_{\mathbb{R}^3 \setminus \Omega} + c(\varepsilon) \chi_{\Omega}) \operatorname{div}(\chi_{\mathbb{R}^3 \setminus \Omega} + d(\varepsilon) \chi_{\Omega}) \operatorname{grad}.$$

Here, Ω is an open, bounded and connected domain of class $\mathcal{C}^2$, the functions $\varepsilon \rightarrow c(\varepsilon)$, $\varepsilon \rightarrow \rho(\varepsilon)$ are real valued when $\varepsilon \in \mathbb{R}$ and analytic in a complex neighbourhood of the origin. We assume that

$$c(\varepsilon) = c_o + c_1 \varepsilon + \mathcal{O}(\varepsilon^2), \quad d(\varepsilon) = \frac{\lambda_o - \rho(\varepsilon) - 1/2}{\lambda_o - \rho(\varepsilon) + 1/2}, \quad \rho(\varepsilon) = \rho_o \varepsilon + \mathcal{O}(\varepsilon^2),$$

with: $c_o > 0$, $c_1, \rho_o \neq 0$, while λ_o is a simple eigenvalue of the Neumann-Poincaré operator on $\partial\Omega$.

The selfadjoint definition of $L(0)$ is provided through sign-indefinite transmission conditions at $\partial\Omega$. Such operators are used as models of electromagnetic materials with sign-changing coefficients. It is known that the wave-propagation through this type of material interfaces exhibits, in the quasi-static regime, localization phenomena referred to as plasmonic resonances, see [1]. Sign-indefinite Laplacians ($c(\varepsilon) = 1$, $d(\varepsilon) < 0$ in our setting) were considered in bounded domains in [2] and [3]. A limiting absorption principle and the well-posedness in the more general case of not constant contrast were obtained in [6] using complementing boundary conditions.

Following the line of [4] and [5], I will recall how these models can be considered in the framework of the mixed perturbation theory of selfadjoints. I will show that $L(0)$ has an eigenvalue at $\kappa = 0$, which, in turns, explains the localization of the scattering solution in the quasi-static regime. Then, using an analytic implicit-function argument and a Lyapunov-Schmidt reduction, I will show that, under the analytic perturbation of the coefficients, such zero-eigenvalue splits into a couple of poles $\kappa_{\pm}(\varepsilon)$ of the Q -function of $(L(\varepsilon), -\Delta)$. In particular, for $\varepsilon \in (0, \varepsilon_o)$ and depending on the sign of ρ_o/λ_o , the points $\kappa_{\pm}(\varepsilon)$ are either a couple of resonances of $(L(\varepsilon), -\Delta)$, or an eigenvalue and a resonance.

Work in collaboration with A. Posilicano.

References

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