

COMPUTER-ASSISTED CLASSIFICATION OF THE UNSTABLE SPECTRUM OF A SELF-SIMILAR BLOWUP SOLUTION

Michael **PLUM**

Karlsruhe Institute of Technology
Karlsruhe, Germany
michael.plum@kit.edu

Abstract

We study a nontrivial self-similar blowup solution of a focusing energy-supercritical semilinear wave equation in three space dimensions with seventh-power nonlinearity. The spectrum of the linearization at this solution is a central object in the analysis of the corresponding dynamics. The self-similar profile is not available in closed form. We first enclose it rigorously by a computer-assisted fixed-point argument around an explicit high-order approximation. The analyticity of the profile required for the subsequent spectral analysis is then derived from the nonlinear profile equation itself. For the spectral problem, the original first-order system is reduced to a scalar singular differential equation and then transformed to a half-line Schrödinger equation. A Jost solution (with "regular" behaviour at $+\infty$) is constructed through a Volterra integral equation, and by a shooting technique a scalar characteristic function is obtained whose zeros are exactly the eigenvalues in the relevant half-plane. Rigorous Volterra estimates and interval computations exclude all eigenvalues in the closed strip $0 \leq \operatorname{Re}(\lambda) \leq \frac{2}{3}$. In the region $\operatorname{Re}(\lambda) > \frac{2}{3}$, self-adjointness of the half-line Schrödinger problem rules out non-real eigenvalues, and Sturm-type zero counts determine the real spectrum. The final classification yields exactly two simple real eigenvalues in the closed right half-plane. One is the time-translation mode and therefore physically meaningless; the second is a genuine unstable mode. Thus, precisely one unstable spectral direction remains, resulting in a codimension-one stable manifold for the original nonlinear problem.