University of Hradec Králové
Faculty of Science

PROCEEDINGS OF
4^{th} INTERNATIONAL PHYSICS DAYS
of HRADEC KRÁLOVÉ 2025

Published on October 20, 2025.


HKIPD
University of Hradec Králové
Faculty of Science
4th Hradec Králové
International PHYSICS DAYS
University of Hradec Králové
9 – 11 October 2025
HONORARY CHAIR
Jan Kříž
Rector of University of Hradec Králové
SCIENTIFIC ADVISORY BOARD
• Bilel Hamil
• Bobomurat Ahmedov
• Izzet Sakalli
• Jutta Kunz
• Miloslav Znojil
• Piero Nicolini
• Robert Mann
• Roman Konoplya
ORGANIZING COMMITTEE
• Andrii Khrabustovskyi
• Bekir Can Lütfüoglu
• Bilel Hamil
• Erdinç Ulaş Saka
• Filip Studnička
INFORMATION
• Day 1 (live only): Talks dedicated to mathematical physics
• Day 2 (live only): Talks dedicated to black hole physics
• Day 3: Live or pre-recorded talks on any topic in theoretical physics (available on demand)
• Deadline for titles, abstracts & pre-recorded talks: 8/10/ 2025
• Speakers of pre-recorded talks must be online during their session, otherwise the talk will not be played.
• All presentations via Zoom – link will be shared on 9/10/2025
Selected papers will be peer-reviewed and published in the International Journal of Gravitation and Theoretical Physics (IJGTP)
uni.uhk.cz/ipd/

Conference Programme

Prague, October 9–11

Day 1: Mathematical Physics (October 9)

Time (CET)	Speaker (Affiliation)	Title
09:00–09:15	*Opening*	
09:15–10:00	Pavel Exner (Prague)	Discrete spectrum induced by geometric perturbations of soft waveguides
10:00–10:30	Jussi Behrndt (Graz)	Weak coupling for Schrödinger operators with complex potentials
10:30–11:00	*Coffee break*	
11:00–11:45	Jari Taskinen (Helsinki)	Linear heat equation in periodic domains
11:45–12:15	Renata Bunoiu (Metz)	On the homogenization of Maxwell's equations with sign-changing coefficients
12:15–13:30	*Lunch break*	
13:30–14:00	Rostyslav Hryniv (Lviv)	Scattering on the line decorated by compact quantum graphs
14:00–14:30	Julia Orlik (Kaiserslautern)	Homogenization of Maxwell's equations in a thin periodic layer to study Faraday Cage-like effect
14:30–15:00	Vladimir Lotoreichik (Prague)	Shape optimization with large magnetic fields in two dimensions
15:00–15:30	*Coffee break*	
15:30–16:00	Yuriy Golovatyi (Lviv)	Quantum graphs with Coulomb-type potentials
16:00–16:30	Olaf Post (Trier)	Different variants of generalised operator norm convergence
16:30–17:00	Ian Wood (Kent)	Interface problems for non-selfadjoint Maxwell equations

Day 2: Black Hole Physics (October 10)

Time (CET)	Speaker (Affiliation)	Title
08:30–09:00	Jan Novák (Czech Technical University (Czech Republic))	The ring paradigm and the old problem of the cosmological constant
09:00–09:30	Angel Rincón (Silesian University in Opava (Czech Republic))	Perturbations of a Quantum-Inspired Black Hole in a Five-Dimensional de Sitter Background
09:30–10:00	Thomas D. Pappas (Silesian University in Opava (Czech Republic))	Dirty Black Holes, Clean Signals: Near-Horizon vs. Environmental Effects on Grey-Body Factors and Hawking Radiation
10:00–10:30	Abdellah Touati (University of Bouira (Algeria))	Temperature and phase transitions of a deformed Schwarzschild black hole in non-commutative gauge theory of gravity
10:30–11:00	*Coffee break*	
11:00–11:30	Bobur Turinov (Ulugh Beg Astronomical Institute (Uzbekistan))	Energetic Properties of Black holes in Scalar-Tensor-Vector Gravity
11:30–12:00	Alfio Maurizio Bonanno (University of Catania (Italy))	Regular Black hole geometries from proper time flow equations
12:00–12:30	Ahmadjon Abdujabbarov (Ulugh Beg Astronomical Institute (Uzbekistan))	Black hole shadow as useful tool for testing the gravity theories
12:30–13:00	Bobomurat Ahmedov (Uzbekistan Academy of Sciences (Uzbekistan))	Current Status of Relativistic Astrophysics in Uzbekistan
13:00–13:30	*Lunch break*	

Time (CET)	Speaker (Affiliation)	Title
13:30–14:00	David Kubizňák (Charles University (Czech Republic))	Excising Cauchy Horizons with Nonlinear Electrodynamics
14:00–14:30	Robie Hennigar (Durham University (United Kingdom))	Regular Black Holes from Pure Gravity
14:30–15:00	Athanasios G. Tzikas (University of Bergamo (Italy))	Quantum gravity black holes as dark matter
15:00–15:30	Salvatore Samuele Siretta (Italy)	Stellar object from Quantum Gravity
15:30–16:00	*Coffee break*	
16:00–16:30	Alexander Zhidenko (Universidade Federal do ABC (Brazil))	Primary hairs may create echoes
16:30–17:00	Adailton Azevedo Araújo Filho (Universidade Federal da Paraíba (Brazil))	Influence of a Kalb–Ramond black hole on neutrino behavior
17:00–17:30	Julio Oliva Zapata (Universidad de Concepción (Chile))	Phase Transitions, Quasinormal Modes, and Holographic Operator Potential of Regular Black Holes in AdS
17:30–18:00	Ghulam Mustafa (Zhejiang Normal University (China))	Some relativistic properties of wormhole
18:00–18:30	Alexander Vikman (Czech Academy of Sciences (Czech Republic))	Domain Walls and Gravitational Waves

Day 3: Contributed Talks (October 11)

Time (CET)	Speaker (Affiliation)	Title
Live Talks		
09:00–09:15	Collins Okon Edet (Universiti Malaysia Perlis (Malaysia))	Tunable Quantum Thermal Machines
09:15–09:30	Precious Amadi (Universiti Malaysia Perlis (Malaysia))	Spectral Amplitude Coded–OCDMA OCP for Multiple Access Network
09:30–09:45	Abubakir Shermatov (National Research University TIIAME (Uzbekistan))	QPOs analyses and circular orbits of charged particles around magnetized black holes in Bertotti–Robinson geometry
09:45–10:00	Kenza Zablak (Ecole Normale Supérieure, Algiers (Algeria))	Calculation of Radiative Corrections of the Loop for the fermionic field for the Gauge Field Propagator of Noncommutative Gauge Supersymmetric Theory
10:00–10:15	Amel Ait El Djoudi* (Ecole Normale Supérieure–Kouba, Algiers (Algeria))	Investigating the sound velocity in the mixed hadronic–QGP system for nucleons and pions in the hadronic phase
10:15–10:30	Djamel Eddine Mouhata Raber (Université de Kasdi Merbah–Ouargla (Algeria))	Klein–Gordon Oscillator Using Generalized Dunkl Derivatives in the Path Integral Formalism
10:30–10:45	Mouna Bouhelal (LPAT, Echahid Cheikh Larbi Tebessi University (Algeria))	Insights into Mirror Nuclei Structure: A Theoretical Spectroscopic Study of ^{21}O and ^{21}Al
10:45–11:00	Abir Selim (LPAT, Echahid Cheikh Larbi Tebessi University, Tebessa (Algeria))	Theoretical Exploration of the Structure of the Mirror Nuclei ^{21}F and ^{21}Al
11:00–11:30	*Coffee break*	

* Speaker did not present the talk.

Time (CET)	Speaker (Affiliation)	Title
Pre-recorded Talks		
11:30–11:45	Ali Akyüz (Burdur Mehmet Akif Ersoy University (Turkey))	Interactive Simulation of Ultrasonic Cavitation Dynamics
11:45–12:00	Muzaffer Erdoğan (Tekirdağ Namık Kemal University Physics Department (Turkey))	Analytical Calculation of Mutual Inductance between Laterally Misaligned Circular Coils in Wireless Power Transfer Systems
12:00–12:15	Zeyneb Tabli (University of Hassiba Benbouali (Algeria))	Exact Nonlinear Solutions of the Sasa–Satsuma Equation
12:15–12:30	Meriem Abdelaziz (University of Biskra (Algeria))	Analytical Study of Quantum Deformation Effects on C_6H_6 and B_4H Molecules in (Anti)-de Sitter Space
12:30–12:45	Zohra Mehri* (Ahmed Zabana University of Relizane (Algeria))	Disorder-Induced Anderson Localization of Expanding Quantum Droplets
12:45–13:00	Mustapha Anis Younes (Université de Bejaia (Algeria))	A Three-Party Lightweight Quantum Key Distribution Protocol in a Restricted Quantum Environment
13:00–13:15	Lakhdar Sek (University of El Oued (Algeria))	Thermal properties of the Dirac oscillator in noncommutative phase space
13:15–13:30	Narges Heidari (Center for Theoretical Physics, Khazar University (Azerbaijan))	Effect of Noncommutative Geometry on Optical Appearance of a Regular Black Hole
13:30–13:45	Bekzod Rahmatov (SamSU (Uzbekistan))	Fundamental frequencies and thermodynamical properties of black holes in the presence of exotic matter
13:45–14:00	Hadj Mouloudj* (Ecole Normale Supérieure–Kouba / University of Chlef (Algeria))	Microscopic analysis of the mean square charge radius $\langle r^2 \rangle$ in even–even cerium (Ce) isotopes, based on deformation energies and deformation parameters
14:00–14:15	Amel Merrad (Hassiba Benbouali University of Chlef (Algeria))	Cs:AlCuI: A New Class of Lead-Free Double Perovskites with Promising Structural, Mechanical, Electronic and Optical Properties
14:00–14:15	Atika Mehedi* (Hassiba Benbouali University of Chlef (Algeria))	Two-component Bose gases at ultra-cold temperature

* Speaker did not present the talk.

ORGANIZING COMMITTEE

- Bekir Can LÜTFÜOGLU (Czechia) (Conference Chair)
- Andrii KHRABUSTOVSKYI (Czechia)
- Bilel HAMIL (Algeria)
- Erdinç Ulaş SAKA (Turkey)
- Filip STUDNIČKA (Czehia)

Contents

DISCRETE SPECTRUM INDUCED BY GEOMETRIC PERTURBATIONS OF SOFT WAVEGUIDES

Pavel **EXNER**

Doppler Institute for Mathematical Physics and Applied Mathematics
Prague, Czech Republic
exner@ujf.cas.cz

Abstract

The talk topic is bound states coming from geometric perturbations of systems that may regarded as a soft version of a straight quantum waveguides. We focus on two particular examples. One is a two-dimensional Schrödinger operator with the potential in the form of a channel of fixed profile built over a curve which is not straight but it is straight outside a compact. The second one concerns a sort of soft waveguide with a longitudinal periodic structure, specifically an array of potential wells. Here we again consider perturbations which preserve the array structure outside a compact region and in this case we allow for higher dimensions. We also mention some open problems.

WEAK COUPLING FOR SCHRÖDINGER OPERATORS WITH COMPLEX POTENTIALS

Jussi **BEHRNDT**

Institut für Angewandte Mathematik, Technische Universität Graz
Graz, Austria
behrndt@tugraz.at

Abstract

We study the discrete eigenvalues emerging from the threshold of the essential spectrum of a one- or two-dimensional Schrödinger operator with complex-valued decaying potential in a weak coupling regime. This talk is based on joint work with M. Holzmann, P. Siegl, and N. Weber.

LINEAR HEAT EQUATION IN PERIODIC DOMAINS

Jari **TASKINEN**

Department of Mathematics and Statistics, University of Helsinki
Helsinki, Finland
jari.taskinen@helsinki.fi

Abstract

We treat the classical linear heat equation in unbounded, periodic waveguides $\Pi \subset \mathbb{R}^d$ by using the Floquet-Bloch transform methods. As well known, the behavior of the solution of the heat equation is determined by the underlying spectral Laplace problem. The Floquet-Bloch transform F turns the elliptic problem on the unbounded domain Π into a corresponding elliptic model problem on the bounded periodic cell ϖ of Π. The Floquet-Bloch transform can also be directly applied to the original heat equation, which yields a heat equation with mixed boundary conditions on the periodic cell ϖ. We analyse the connection between the two approaches to the problems. In the case of a disjoint component of the essential spectrum, we describe the corresponding spectral projection and note that the translated Wannier functions form an orthonormal basis in $\mathcal{H}_S$. Applications to the heat equation follow immediately.

In the case Π is periodic in d directions, we observe that for a general, integrable initial data, the solution decays for large t at the same rate $t^{-d/2}$ as in the case of the Cauchy problem in the entire Euclidean space, but initial data with certain vanishing conditions for the x-integral leads to faster decay rates: given any positive integer N, we find sufficient conditions for the initial data so that the solution decays at least at the rate t^{-N} .

References

[1] M. Rosenberg, J. Taskinen, Some aspects of the Floquet theory for the heat equation in a periodic domain, J. Evolution Equations 24 (2024), 23.

[2] M. Rosenberg, J. Taskinen, Heat equation in a periodic domain with special initial data, J. Differential Equations 451 (2026).

ON THE HOMOGENIZATION OF MAXWELL'S EQUATIONS WITH SIGN-CHANGING COEFFICIENTS

Renata **BUNOIU**

IECL, CNRS, University of Lorraine
Metz, France
renata.bunoiu@univ-lorraine.fr

Abstract

In this presentation, we focus on the homogenization of Maxwell's equations in a composite medium containing small, periodically distributed inclusions made of a negative material, i.e., a material with negative permittivity and permeability. Due to the sign change of the coefficients, it is not straightforward to obtain uniform energy estimates and apply classical homogenization techniques. Our analysis is based on the study of two scalar problems for which we obtain a criterion based on the physical parameters guaranteeing the uniform invertibility of the associated operators as the size of the inclusions tends to zero. These results obtained for the scalar problems are then used to obtain uniform energy estimates for the Maxwell system. This requires resolving an additional difficulty related to the indefinite nature induced by the frequency term, which we achieve by obtaining a uniform compactness-type result.

The results presented are based on the common work in collaboration with Lucas Chesnel (Inria, Ensta Paris), Karim Ramdani (Inria Nancy) and Mahran Rihani (SNCF Réseaux).

References

[1] R. Bunoiu, L. Chesnel, K. Ramdani, M. Rihani, Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients, Annales de la Faculté des Sciences de Toulouse: Mathématiques, Ser. 6, 30 (5), 1075–1119, 2021.

SCATTERING ON THE LINE DECORATED BY COMPACT QUANTUM GRAPHS

Rostyslav **HRYNIV**

Faculty of Applied Sciences, Ukrainian Catholic University
Lviv, Ukraine

Faculty of Natural Sciences, University of Rzeszów
Rzeszów, Poland

rhryniv@ucu.edu.ua

Abstract

Scattering of quantum particles on graphs has been actively studied since the late 1980s [1], and the scattering theory on the line has by now been successfully extended to quantum graphs. In this talk, we address the scattering problem on a line decorated with a locally periodic array of compact quantum graphs (see Fig. 1). These decorations act as resonant scatterers, effectively inducing energy-dependent point interactions (pseudo-potentials) on the line. Remarkably, even in the absence of any edge potential, such systems can display highly non-trivial transmission characteristics such as tunneling effects and the emergence of effective band-pass filters (see Fig. 2), similarly to the scattering on δ'-combs [2].

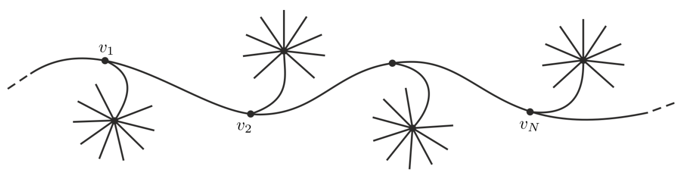

Figure 1: Locally periodic graph Γ decorated with dandelions.

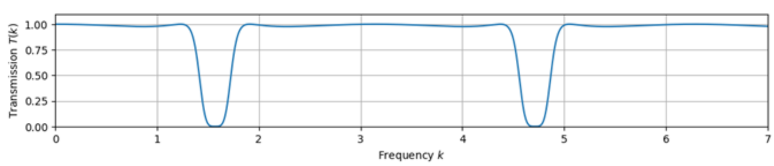

Figure 2: Transmission probabilities $T(k)$ for three single-edge decorations

The model is exactly solvable and allows great flexibility in terms of the choice of compact graph, the number of decorations, and the interface conditions at the junction points. This flexibility can enable analytic construction of scattering models with desired transport properties, thus providing a rigorous framework for exploring wave propagation in structured quantum systems.

The talk is based on the joint work with Khrystyna Buhrii and Yuriy Golovaty [3].

References

[1] Gerasimenko, N.I. and Pavlov, B.S. Scattering problems on noncompact graphs. *Theoret. and Math. Phys.*, **74** (1988), no. 3, 230–240.

[2] Golovaty, Y., Hryniv, R., and Lavrynenko, S. Transmission resonances in scattering by δ'-like combs. *J. Phys. A: Math. Theor.* **58** (2025), id. 275304 (21 p.).

[3] Buhrii, Kh., Golovaty, Y., and Hryniv, R., Scattering on locally periodic quantum graphs. *Submitted.*

HOMOGENIZATION OF MAXWELL'S EQUATIONS IN A THIN PERIODIC LAYER TO STUDY FARADAY CAGE-LIKE EFFECT

Julia **ORLIK**

Fraunhofer Institute for Industrial Mathematics
Kaiserslautern, Germany
julia.orlik@itwm.fraunhofer.de

Abstract

This work considers the Maxwell equations for two domains separated by a heterogeneous layer. The layer has a δ-periodic structure in the $x_1 - x_2$ plane and is cylindrical in the x_3 direction, where $\delta << 1$. The layer consists of a one-connected component (metallic layer) and a collection of disconnected regions. The connected part is made of highly conductive material, while the disconnected regions are filled with air or a non-conductive material. The main difficulty arises due to the purely real-valued permittivity coefficient in the disconnected regions of the layer, while in the metallic part, the complex part of the permittivity coefficient is non-zero parameter, ε. We have considered several cases for the pair (δ, ε), and studied the asymptotic behavior of the considered Maxwell system and three different types of limit interface problems: partial shielding, full shielding, and no shielding.

This is a joint work with S. Aiyappan, Georges Griso and Abu Sufian.

SHAPE OPTIMIZATION WITH LARGE MAGNETIC FIELDS IN TWO DIMENSIONS

Vladimir **LOTOREICHIK**

Department of Theoretical Physics, Nuclear Physics Institute, Czech Academy of Sciences
Řež, Czech Republic
lotoreichik@ujf.cas.cz

Abstract

In this talk, we will demonstrate that in the limit of strong magnetic fields, the optimal domains for eigenvalues of magnetic Laplacians tend to exhibit symmetry. We establish several asymptotic bounds on magnetic eigenvalues to support this conclusion. Our main result implies that if, for a bounded simply-connected planar domain, the n-th eigenvalue of the magnetic Dirichlet Laplacian with uniform magnetic field is smaller than the corresponding eigenvalue for a disk of the same area, then the Fraenkel asymmetry of that domain tends to zero in the strong magnetic field limit. Comparable results will also be discussed for the magnetic Dirichlet Laplacian on rectangles, as well as the magnetic Dirac operator with infinite mass boundary conditions on smooth domains.

These results are obtained in collaboration with Léo Morin.

QUANTUM GRAPHS WITH COULOMB-TYPE POTENTIALS

Yuriy **GOLOVATY**

Faculty of Mechanics and Mathematics, Ivan Franko National University of Lviv
Lviv, Ukraine

Faculty of Applied Sciences, Ukrainian Catholic University
Lviv, Ukraine

yuriy.golovaty@lnu.edu.ua

Abstract

In recent decades, the theory of differential operators on metric graphs has been the subject of extensive and systematic study, particularly due to its numerous applications in solid-state physics and engineering. From a physics standpoint, however, the most exciting application of this theory is quantum graphs. Quantum graphs provide effective mathematical models that allow us to study quantum systems in a framework that makes explicit solutions possible.

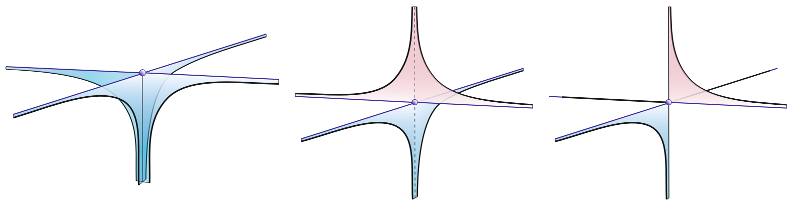

Figure 1: The "classic" and exotic Coulomb potentials.

In this talk, we will construct exactly solvable models of quantum graphs with Coulomb-type potentials whose singularities are located at the vertices (see Fig. 1). This problem generalizes the well-known problem of constructing a one-dimensional model of a hydrogen atom [1]. We describe all self-adjoint realizations of formal Coulomb Hamiltonians on a star graph. Regularizing these potentials answers the question of which vertex interactions are physically motivated for the Coulomb potentials. We establish conditions under which Schrödinger operators with cut-off Coulomb potentials coupled with $(\alpha\delta' + \beta\delta)$-like ones converge in the norm resolvent topology. The 1D Coulomb potentials and the δ'-pseudopotential are very sensitive to their regularization method. In particular, the one-dimensional model of the hydrogen atom shows a critical dependence on the behaviour of cut-off Coulomb potentials at small

distances [2]. These properties are also inherent in quantum graphs with Coulomb potentials.

This talk is based on work [3].

References

[1] Loudon, R. One-dimensional hydrogen atom. Proc. R. Soc. A Math. Phys. Eng. Sci. 472(2185), 20150534 (2016).

[2] Golovaty, Y. 1D Schrödinger operators with Coulomb-like potentials. J. Math. Phys. 60(8), 082105 (2019).

[3] Golovaty, Y. Quantum graphs: Coulomb-type potentials and exactly solvable models. Ann. Henri Poincaré 24, 2557-2585 (2023).

DIFFERENT VARIANTS OF GENERALISED OPERATOR NORM CONVERGENCE

Olaf **POST**

Fachbereich 4–Mathematik, Universität Trier
Trier, Germany
olaf.post@uni-trier.de

Abstract

In this talk, we present different concepts of how to compare the distance in operator norm of operators acting in different Hilbert spaces. We establish equivalence of the distances in certain situations and compare it with some spectral distance. We also introduce a related notion of convergence, applicable for resolvents of operators acting in varying Hilbert spaces.

This is a joint work with Sebastian Zimmer, Trier.

INTERFACE PROBLEMS FOR NON-SELFADJOINT MAXWELL EQUATIONS

Jan **WOOD**

School of Mathematics, Statistics and Actuarial Sciences, University of Kent
Canterbury, UK
I.Wood@kent.ac.uk

Abstract

In this talk, we consider the spectrum of a non-selfadjoint operator pencil generated by the time-harmonic Maxwell problem with a flat interface between two dispersive media.

The dependence on the spectral parameter, i.e. the frequency, appears in the dielectric function and we make no assumptions on its form. In particular, to model dispersive media, the dielectric function is allowed to be complex, yielding a non-selfadjoint problem.

We consider the different types of essential spectrum arising in this model. We will concentrate on some simple examples where the spectrum can be fairly explicitly determined - such as periodic or homogeneous media in each half-space.

This is joint work with Malcolm Brown (Cardiff), Tomas Dohnal (Halle), Karl Michael Schmidt (Cardiff) and Michael Plum (Karlsruhe).

THE RING PARADIGM AND THE OLD PROBLEM OF THE COSMOLOGICAL CONSTANT

Jan **NOVÁK**

Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University,
Trojanova 13, Praha 2, 166 07, Czech Republic
Orcid number: 0000-0002-5943-4052
jan.novak@johnynewman.com

Abstract

We formulate an approach to quantum gravity, called the ring paradigm. Gravity is mediated superluminally, and the graviton is described as a phonon on the grid of matter in the Universe. This theory has very interesting applications to cosmology and would give us a first hint on how to solve the old problem of the cosmological constant. It further comes with new impulses to the scalar field theories because the gravitational ring decays to some phantom field.

PERTURBATIONS OF A QUANTUM-INSPIRED BLACK HOLE IN A FIVE-DIMENSIONAL DE SITTER BACKGROUND

Ángel **RINCÓN**

Research Centre for Theoretical Physics and Astrophysics, Institute of Physics, Silesian University in Opava
Bezručovo náměstí 13, CZ-746 01 Opava, Czech Republic
Orcid number: 0000-0001-8069-9162
angeles19531322@gmail.com

Abstract

This talk explores the quasinormal modes (QNMs) of a five-dimensional scale-dependent black hole with a positive cosmological constant, examining scalar, electromagnetic, and Dirac perturbations. We derive exact analytical solutions within the scale-dependent gravity framework, selecting a modified solution family, and analyze the effective potentials and lapse function for each perturbation type. The WKB semi-analytical method is employed to compute QNMs, focusing on a small range of the scale-dependent parameter ε and using a first-order approximation for the lapse function, valid for small ε. QNM frequencies are calculated for both the classical case and non-zero ε, with results presented in tables and figures. The findings confirm the stability of the black hole across all perturbation types, as indicated by the negative imaginary parts of the QNM frequencies.

DIRTY BLACK HOLES, CLEAN SIGNALS: NEAR-HORIZON VS. ENVIRONMENTAL EFFECTS ON GREY-BODY FACTORS AND HAWKING RADIATION

Thomas D. **PAPPAS**

Research Centre for Theoretical Physics and Astrophysics, Institute of Physics, Silesian University in Opava
Bezručovo náměstí 13, CZ-746 01 Opava, Czech Republic
Orcid number: 0000-0003-2186-357X
thomas.pappas@physics.slu.cz

Abstract

Grey-body factors are not only essential ingredients for computing the intensity of Hawking radiation, but also serve as characteristics of black hole's geometry that are closely related to its quasinormal modes. Importantly, they tend to be more stable under small deformations of the background spacetime. In this work, we carry out a detailed analysis of grey-body factors and Hawking radiation for a spherically symmetric black hole subject to localized deformations that do not alter the Hawking temperature: near-horizon modifications to simulate possible new physics or matter fields, and far-zone perturbations to model environmental or astrophysical effects. We show that environmental deformations have only a minor impact on the grey-body factors and Hawking radiation–unless the additional potential barrier created by the environment becomes comparable in height to the primary peak associated with the black hole itself, a scenario more relevant to nonlinear dynamics. In contrast, near-horizon deformations significantly affect the Hawking spectrum, particularly in the low-frequency regime.

TEMPERATURE AND PHASE TRANSITION OF A DEFORMED SCHWARZSCHILD BLACK HOLE IN NON-COMMUTATIVE GAUGE THEORY OF GRAVITY

Abdellah **TOUATI**

Department of Physics, Faculty of Exact Sciences, University of Bouira
10000-bouira, Bouira, Algeria
Orcid number: 0000-0003-4478-2529
touati.abph@gmail.com

Slimane **ZAIM**

Department of Physics, Faculty of Sciences of Matter, University of Batna-1
05000-batna, Batna, Algeria
zaim69slimane@yahoo.com

Abstract

In this study, we investigate the Hawking temperature and phase transitions of a deformed Schwarzschild black hole (SBH) within the non-commutative (NC) gauge theory of gravity (GTG). First, we compute the leading-order NC corrections (in the parameter Θ) to the metric components using the star product and the Seiberg-Witten map (SW), taking into account both the (r, θ) and (r, ϕ) twists. Then, the temperature of this black hole is obtained using the NC event horizon with a correction up to the second order in Θ. The obtained results show that the non-commutativity removes the divergence behavior of the temperature, and makes a difference in the poles-equator temperature. Also, the estimation of the NC parameter shows a new fundamental length at the Planck scale $\Theta^{Phys} \sim 10^{-35}m$, which indicates that spacetime is quantized at the Planck level. Finally, a simple analysis to the heat capacity show a phase transition of the NC SBH, and predicts a new evaporation scenario. Also, this geometry prevent SBH from a complete evaporation leading to the emergence of a stable BH remnant in the final stage of evaporation is emerged.

ENERGETIC PROPERTIES OF BLACK HOLES IN SCALAR-TENSOR-VECTOR GRAVITY

Bobur **TURIMOV**

Ulugh Beg Astronomical Institute,
33 Astronomicheskaya str., Tashkent, 100052, Uzbekistan
Orcid number: 0000-0003-1502-2053
bturimov@astrin.uz

Abstract

The analysis of the gravitational field surrounding a Kerr-MOG black hole reveals that it possesses a stronger gravitational field, a larger event horizon, and can rotate faster than a standard Kerr black hole due to the influence of Scalar–Tensor–Vector Gravity (STVG). We examined the impact of STVG on the circular motion of massive particles around the Kerr-MOG black hole and analyzed the characteristics of the Innermost Stable Circular Orbit (ISCO) for such particles. The results indicate that STVG significantly affects the efficiency of energy extraction from a rotating black hole, allowing for efficiencies exceeding 100% through the Penrose process. Additionally, we investigated the gravitational synchrotron radiation analogue emitted by a massive particle orbiting a Kerr-MOG black hole and demonstrated that STVG alters the intensity of gravitational radiation from binary systems involving stellar-mass and supermassive black holes.

REGULAR BLACK HOLE GEOMETRIES FROM PROPER TIME FLOW EQUATIONS

Alfio Maurizio **BONANNO**

INFN, Sezione di Catania
via S. Sofia 64, I-95123, Catania, Italy.
INAF, Osservatorio Astrofisico di Catania,
via S.Sofia 78, I-95123 Catania, Italy.
Orcid number: 0000-0003-3175-9776
alfio.bonanno@inaf.it

Abstract

In this talk, I will present a class of regular black hole geometries derived from the proper-time renormalization group approach to asymptotically safe gravity [1]. A key question in this framework concerns the robustness of physical predictions with respect to the choice of regularization scheme. I will address this issue by computing several important observables for the resulting quantum-corrected black holes, which are non-singular and asymptotically Schwarzschild.

The quasinormal mode spectrum shows clear deviations from the classical case, while the Hawking radiation spectrum is significantly suppressed, indicating a slower evaporation rate and weaker constraints on primordial black holes as dark matter candidates. By contrast, shadows and ISCO radii remain compatible with observational data. These findings demonstrate that singularity resolution and its main observational signatures are robust physical outcomes, rather than artifacts of the regularization scheme.

References

[1] A. Bonanno, R. A. Konoplya, G. Oglialoro, A. Spina, Submitted to JCAP, arXiv:2509.12469 [gr-qc].

BLACK HOLE SHADOW AS USEFUL TOOL FOR TESTING THE GRAVITY THEORIES

Ahmadjon **ABDUJABBAROV**

Ulugh Beg Astronomical Institute, Uzbekistan Academy of Sciences
Astronomy Str. 33, Tashkent 100052, Uzbekistan
Orcid number: 0000-0002-6686-3787
ahmadjon@astrin.uz

Abstract

The observation of black hole shadows by the Event Horizon Telescope (EHT) has opened a new window for testing gravity theories in the strong-field regime. The shadow, a dark region caused by photon capture near the event horizon, encodes information about spacetime geometry, making it a powerful probe for alternatives to general relativity (GR). By comparing theoretical predictions of shadow morphology – influenced by parameters such as spin, charge, and modified gravity corrections – with high-resolution imaging data, stringent constraints can be placed on deviations from GR, including scalar-tensor, f(R), and extra-dimensional theories. This review highlights recent advances in shadow of the black holes, the role of gravitational physics, and the coordinate independent way to describe the image.

CURRENT STATUS OF RELATIVISTIC ASTROPHYSICS IN UZBEKISTAN

Bobomurat J. **AHMEDOV**

Ulugh Beg Astronomical Institute, Uzbekistan Academy of Sciences, Tashkent 100052, Uzbekistan
Institute for Advanced Studies, New Uzbekistan University, Movarounnahr str. 1, Tashkent 100000, Uzbekistan
Orcid number: 0000-0002-1232-610X
ahmedov@astrin.uz

Abstract

The past decade has witnessed the rapid development of relativistic astrophysics in Uzbekistan, centered at the Laboratory of Theoretical Astrophysics of the Ulugh Beg Astronomical Institute (UBAI). This talk will survey the current research landscape, detailing the group's principal scientific directions. Key areas of focus include:

Theoretical Modeling: Analytical and numerical studies of accretion processes, magnetohydrodynamic (MHD) jet formation, and particle dynamics in the vicinities of supermassive and stellar-mass black holes.

Compact Objects: Investigation of the solutions of the gravitational field equations, electrodynamics of magnetised neutron stars and the energetics of gravitational compact objects.

Observational Tests of GR: Utilizing data from gravitational wave observatories (LIGO/Virgo) and Event Horizon Telescope (EHT) collaborations to test General Relativity in strong-gravity environments.

I will emphasize the synergy between theoretical work and multi-messenger data analysis, underscoring the laboratory's integration into international scientific collaboration. This presentation will illustrate how Uzbekistan astrophysicists are actively advancing the frontiers of knowledge in one of modern astrophysics' most dynamic fields.

EXCISING CAUCHY HORIZONS WITH NONLINEAR ELECTRODYNAMICS

David **KUBIZŇÁK**

Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University,
V Holešovickách 2, 180 00 Prague 8, Czech Republic
0000-0003-0683-4578
david.kubiznak@matfyz.cuni.cz

Abstract

Charged and/or rotating black holes in General Relativity feature Cauchy horizons, which indicate a breakdown of predictability in the theory. Focusing first on spherically symmetric charged black holes, we remark that the inevitability of Reissner-Nordström Cauchy horizon is due to the divergent electromagnetic self-energy of point charges. We demonstrate that any causal theory of nonlinear electrodynamics that regularizes the point charge self-energy also eliminates Cauchy horizons for weakly charged black holes. These black holes feature one (event) horizon and a spacelike singularity, analogous to the Schwarzschild metric. An example with Born-Infeld electrodynamics illustrates how this gives rise to an upper bound on the charge, which we compare with known bounds. Generalization to a rotating case will also be briefly discussed.

REGULAR BLACK HOLES FROM PURE GRAVITY

Robie A. **HENNIGAR**

Centre for Particle Theory, Department of Mathematical Sciences, Durham University,
Durham DH1 3LE, United Kingdom
Orcid number: 0000-0002-9531-6440
robie.a.hennigar@durham.ac.uk

Abstract

I will review a recent program showing that regular (non-singular) black holes arise as generic, exact solutions of gravitational actions with infinite towers of higher-curvature corrections. After introducing the relevant quasi-topological gravities and their key structural properties, I will explain how resummation of these corrections yields black hole geometries that are everywhere regular. I will then outline how this framework addresses dynamical questions, including formation from spherical collapse of matter.

QUANTUM GRAVITY BLACK HOLES AS DARK MATTER

Athanasios **TZIKAS**

Department of Engineering and Applied Sciences, University of Bergamo
Viale Marconi 5 24044 Dalmine, Bergamo, Italy
Orcid number: 0000-0002-0633-3629
athanasios.tzikas@unibg.it

Abstract

The persistent challenge in quantum gravity research has been the scarcity of experimentally detectable signals. Here, we show a new observational pathway to probe quantum gravity effects. Contrary to previous claims, the quantum decay of de Sitter space into black hole spacetimes can be significant even after inflation and observable on galactic scales. Using the instanton formalism within the no-boundary proposal for a class of quantum-gravity-improved black hole metrics, we show that de Sitter space decay could result in the production of 10^{60} stable Planck-size black hole remnants within the current Hubble horizon, explaining dark matter. Critically, our analysis provides a novel quantum gravitational mechanism for the direct transformation of dark energy into dark matter, suggesting a fundamental link between these enigmatic cosmic components.

STELLAR OBJECT FROM QUANTUM GRAVITY

Salvatore Samuele **SIRLETTI**

Department of Physics and Earth Sciences University of Ferrara
Via Saragat 1 I-44122 Ferrara, Italy
Orcid number: 0000-0003-3383-3351
salvatore.sirletti@unitn.it

Abstract

It has been shown that the UV finiteness of Superstring Theory can led to the derivation of a family of regular black hole solutions in the gravity-matter decoupling limit. The latter is a regime governed by stringy effects like non-commutativity and T-duality. The most natural realization of a non-local structure inheriting noncommutative geometry effects is the Gaussian profile for the energy density in the relativistic stress tensor. In this talk, we present two interesting regular black hole/compact object alternatives that stem from postulating a smooth transition between a quantum gravity dominated region at the origin, and a corona of degenerate nuclear matter around it. The derivation of the resulting metric allows for the description of a regular horizonless Planckian object and a neutron star with a quantum vacuum at its center.

PRIMARY HAIRS MAY CREATE ECHOES

Alexander **ZHIDENKO**

Center for Mathematics, Computing and Cognition (CMCC) Federal University of ABC (UFABC)
Rua Abolição CEP 09210-180 Santo André, SP, Brazil
Orcid number: 0000-0001-6838-3309
alexander.zhidenko@gmail.com

Abstract

In most scenarios studied so far, the appearance of echoes in the ringdown signal requires modifications external to the black hole itself, such as the presence of matter in the near-horizon region, quantum field clouds, or exotic compact objects like wormholes that effectively introduce additional peaks in the effective potential. We show that echoes can naturally arise in a different setting: black holes endowed with primary Proca-Gauss-Bonnet hair. We demonstrate that the primary hair modifies the effective potential in such a way that a second peak is formed, giving rise to late-time echoes without invoking any external environment or exotic horizon-scale physics. Our results highlight a novel mechanism by which primary hairs alone can leave observable imprints on the ringdown signal of black holes in modified gravity.

INFLUENCE OF A KALB-RAMOND BLACK HOLE ON NEUTRINO BEHAVIOR

Adailton Azevêdo **ARAÚJO FILHO**

Department of Physics, Federal University of Campina Grande
P.O. Box 10071 58429-900 Campina Grande, Paraíba, Brazil
Orcid number: 0000-0002-8790-3944
dilto@fisica.ufc.br

Abstract

In this work, we investigate the consequences of Lorentz symmetry breaking induced by a black hole solution within Kalb-Ramond gravity on neutrino dynamics. The analysis is structured around three key phenomena: the energy output associated with neutrino-antineutrino annihilation, the impact of spacetime geometry on the oscillation phases of neutrinos, and the changes in flavor transition probabilities caused by gravitational lensing effects. To complement the theoretical framework, we also carry out a numerical evaluation of neutrino oscillation probabilities, comparing scenarios of normal and inverted mass orderings within a two-flavor approximation.

PHASE TRANSITIONS, QUASINORMAL MODES, AND HOLOGRAPHIC $Q\bar{Q}$ POTENTIAL OF REGULAR BLACK HOLES IN AdS

Julio **OLIVA**

Department of Physics, University of Concepción
Casilla 160-C, Concepción, Chile
Orcid number:0000-0002-0692-4744
julioolivazapata@gmail.com

Abstract

In this talk, I will show that the purely gravitational $\alpha'^3\mathrm{Weyl}^4$ correction of Type IIB supergravity on $\mathrm{AdS}_5 \times S^5$ can, via a field redefinition, be recast as a series of higher-curvature terms up to order Riem^4, with the following key properties:

1. The graviton propagator in AdS matches that of General Relativity, up to an effective Newton's constant;
2. The field equations on spherically symmetric backgrounds avoid the propagation of the scalar mode that typically plagues higher-curvature theories, thereby admitting a version of Birkhoff's theorem;
3. They allow for the construction of a holographic c-function.

These terms are known as quasitopological Lagrangians. While strong evidence for the existence of such a field redefinition already exists in the literature, here we present its explicit form and explore its consequences. Next, by supplementing the α'^3-corrected theory with an infinite series of higher-curvature terms sharing the above properties, we construct an AdS extension of a family of asymptotically flat regular black holes recently developed in [1]. We then study the phase structure of these black holes in the canonical ensemble, compute the fundamental quasinormal mode of a massless scalar probe, and evaluate the holographic $q\bar{q}$ potential. If time permits, I will also discuss the extended thermodynamics of these black holes.

References

[1] P. Bueno, P. A. Cano, R. A. Hennigar, Phys. Lett. B **861**, 139260 (2025).

SOME RELATIVISTIC PROPERTIES OF WORMHOLE

Ghulam **MUSTAFA**

Department of Physics, Zhejiang Normal University
Jinhua, 321004, People's Republic of China
Orcid number: 0000-0003-1409-2009
gmustafa3828@gmail.com

Abstract

Several newly proposed wormhole solutions obtained through different theoretical approaches should be carefully examined and discussed. In addition, their observational signatures, such as the characteristics of shadows, the behavior of light deflection angles, and various other relativistic effects, also deserve thorough analysis. Such discussions not only provide deeper insights into the physical viability of these solutions but also help establish possible connections with astrophysical observations.

DOMAIN WALLS AND GRAVITATIONAL WAVES

Alexander **VIKMAN**

FZU–Institute of Physics of the Czech Academy of Sciences,
Na Slovance 1999/2, 182 00 Prague 8, Czech Republic
Orcid number: 0000-0003-3957-2068
vikman@fzu.cz

Abstract

I will review the fascinating physics of cosmological domain walls. These classical configurations separate different vacua in field theories with spontaneous violation of discrete symmetries. Such theories often appear in high energy theories beyond the standard model of particle physics. I will discuss potential observational problems associated with cosmological domain walls and different ways to avoid these problems. Moreover, I will show that domain walls are a very promising source of gravitational waves. In particular, the network of the domain walls can be the source of the signal recently observed by NANOGrav collaboration. Furthermore, for different values of parameters, one may detect their gravitational waves in currently planned observational missions like Einstein Telescope, TianQin, Taiji, and LISA. This talk is based on [1–7].

References

[1] I. Dankovsky, S. Ramazanov, E. Babichev, D. Gorbunov, A. Vikman, *Cosmic domain walls on a lattice: illusive effects of initial conditions.* arXiv:2509.25367 [hep-ph].

[2] E. Babichev, I. Dankovsky, D. Gorbunov, S. Ramazanov, A. Vikman, *Biased domain walls: faster annihilation, weaker gravitational waves.* arXiv:2504.07902 [hep-ph].

[3] I. Dankovsky, S. Ramazanov, E. Babichev, D. Gorbunov, A. Vikman, JCAP **02**, 064 (2025).

[4] I. Dankovsky, E. Babichev, D. Gorbunov, S. Ramazanov, A. Vikman, JCAP **09**, 047 (2024).

[5] E. Babichev, D. Gorbunov, S. Ramazanov, R. Samanta, A. Vikman, Phys. Rev. D **108**, 123529 (2023).

[6] E. Babichev, D. Gorbunov, S. Ramazanov, A. Vikman, JCAP **04**, 028 (2022).

[7] S. Ramazanov, E. Babichev, D. Gorbunov, A. Vikman, Phys. Rev. D **105**, 063530 (2022).

TUNABLE QUANTUM THERMAL MACHINES

Collins Okon **EDET**

Department of Mathematical Sciences, Faculty of Intelligent Computing, Universiti Malaysia Perlis
02600 Arau, Perlis, Malaysia
Orcid number: 0000-0001-7762-731X
collinsokonedet@gmail.com

Abstract

Heat transport in quantum systems and circuits is crucial for advancing quantum technologies. In this work, we investigate quantum heat transport in a driven hybrid magnon-photon system coupled to two thermal baths at different temperatures. We demonstrate how system parameters govern the asymmetry of the steady-state heat current and show that external driving controls the heat current direction and enables tuning of heat rectification across its full physical range. These results provide valuable insights for designing quantum thermal machines based on driven magnon-photon platforms [1].

References

[1] C. O. Edet, K. Słowik, N. Ali, O. Abah, (2025). *Driven Magnon-Photon System as a Tunable Quantum Heat Rectifier.* arXiv:2503.06301.

SPECTRAL AMPLITUDE CODED-OCDMA QKD FOR MULTIPLE ACCESS NETWORK

Precious Ogbonda **AMADI**

Institute of Engineering Mathematics, Universiti Malaysia Perlis
02600 Arau, Perlis, Malaysia
Orcid number: 0000-0003-3691-3891
amadiwati@gmail.com

Syed Alwee **ALJUNID**

Faculty of Electronic Engineering & Technology, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
Orcid number: 0000-0003-2739-6220
syedalwee@unimap.edu.my

Norshamsuri **ALI**

Faculty of Electronic Engineering & Technology, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
Orcid number: 0000-0002-9348-0714
norshamsuri@unimap.edu.my

Norshamsuri **ALI** Faculty of Electronic Engineering & Technology, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
Orcid number: 0000-0002-9348-0714
norshamsuri@unimap.edu.my

Mohammad Ammar **SYED**

Faculty of Electronic Engineering & Technology, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
Orcid number: 0000-0002-3630-2583
syammar@studentmail.unimap.edu.my

Rosdisham **ENDUT**

Faculty of Electronic Engineering & Technology, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
Orcid number: 0000-0003-3659-9740
rosdisham@unimap.edu.my

Nursalasawati **RUSLI**

Institute of Engineering Mathematics, Universiti Malaysia Perlis
02600, Arau, Perlis, Malaysia
nursalasawati@unimap.edu.my

Abdulrahman M. **ALHASSAN**

Electrical Engineering, Abu Dhabi University
United Arab Emirates
Orcid number: 0000-0003-1710-8985
nursalasawati@unimap.edu.my

Abstract

We present a theoretical framework and analysis of a hybrid multi-user quantum key distribution (QKD) system utilizing spectral amplitude coding optical code division multiple access (SAC-OCDMA) encoding techniques. By assigning unique optical codes to each user, SAC-OCDMA enables spontaneous, asynchronous data transmission without requiring strict synchronization, making it scalable and flexible for quantum networks. In the architecture, each user's quantum signal, initially prepared as weak coherent pulses and encoded using phase or polarization bases, is further spectrally sliced by a SAC-OCDMA encoder in a zero-based cross-correlation code. The physical impairments, comprising spontaneous Raman scattering, four-wave mixing, and crosstalk, were modeled and analyzed. We report a maximum secret key rate of $\sim 10^5$ bps over a transmission distance of ~ 58 km. Furthermore, our analysis demonstrates that careful selection of launch power, simultaneous users, code weight, and spectral bin width is necessary for optimizing the trade-off between multi-user capacity and security performance. In comparison with wave division multiplexing QKD, which has much better spectral efficiency, but strict channel allocation, the flexibility and asynchronous access to secure communication of our OCDMA-QKD design, with zero cross-correlation coding, is more aligned with the needs of quantum security

QPOS ANALYSES AND CIRCULAR ORBITS OF CHARGED PARTICLES AROUND MAGNETIZED BLACK HOLES IN BERTOTTI-ROBINSON GEOMETRY

Abubakir **SHERMATOV**

Institute of Fundamental and Applied Research, National Research University TIIAME
Address: Kori Niyoziy 39, Tashkent 100000, Uzbekistan
Orcid number: 0009-0009-4044-4507
shermatov.abubakir98@gmail.com

Abstract

I will present recent results on the motion of charged particles around magnetized black holes in the Bertotti–Robinson spacetime, also known as the electromagnetic universe. This exact electrovacuum solution of the Einstein–Maxwell equations allows us to explore how magnetic and gravitational fields jointly affect particle dynamics near black holes. I will discuss how the electromagnetic universe parameter modifies the effective potential, angular momentum, and the innermost stable circular orbits (ISCOs). Furthermore, I will show that by applying the relativistic precession model to this system, one can describe the observed quasi-periodic oscillations (QPOs) in microquasars and galactic centers. Using Markov Chain Monte Carlo analysis, we have constrained the black hole mass and electromagnetic parameters with observational QPO data from sources such as XTE J1550–564 and Sgr A*. These results indicate that the interplay between magnetic fields and the Bertotti–Robinson geometry can provide new insights into strong-gravity astrophysics and future tests of alternative gravity theories.

This talk is based on [1].

References

[1] A. Shermatov, J. Rayimbaev, B.C. Lütfüoğlu, A. Abdujabbarov, S. Sardor, I. Ibragimov, M. Vapayev, B. Kuyliev, Eur. Phys. J. C **85**, 1017 (2025).

CALCULATION OF RADIATIVE CORRECTIONS OF THE LOOP FOR THE FERMIONIC FIELD FOR THE GAUGE FIELD PROPAGATOR OF NONCOMMUTATIVE GAUGE SUPERSYMMETRIC THEORY

Kenza **ZAIBAK**

Departement of Physics, Laboratoire de Physique des Particules et de Physique Statistique, Ecole Normale Supérieure
BP 92 Vieux-Kouba, Algeria
Kzaibak2@gmail.com

Abstract

In this paper, we calculate the contribution of the loop for the fermionic field for the Gauge field propagator of noncommutative Gauge supersymmetric theory in two dimensions in Minkowski space. We have found that this contribution is equal to that of the commutative theory, which gives rise to ordinary UV singularities.

INVESTIGATING THE SOUND VELOCITY IN THE MIXED HADRONIC-QGP SYSTEM FOR NUCLEONS AND PIONS IN THE HADRONIC PHASE

Amal **AIT EL DJOUDI**

Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba
B.P. 92, 16050, Vieux-Kouba, Algiers, Algeria
amal.aiteldjoudi@g.ens-kouba.dz

Faiza **RAMDANI**

Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba
B.P. 92, 16050, Vieux-Kouba, Algiers, Algeria

Rokaya **DJIDA**

Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba
B.P. 92, 16050, Vieux-Kouba, Algiers, Algeria

Abstract

The present work is devoted to the investigation of the behavior of the sound velocity squared in the mixed hadronic-Quark Gluon Plasma (QGP) system, during the thermal deconfinement phase transition from a hadronic phase consisting of nucleons to a QGP phase of u, d and s quarks within their antiquarks and gluons. For this, we use a thermodynamical approach in which we investigate the pressure, the energy density and the related sound velocity squared. The computation of these quantities is done numerically, and their graphical variations are illustrated, with varying temperature and volume at zero chemical potential. The obtained results are analysed and compared to those of our previous works [1–3] carried out for a transition from a pionic hadronic phase to a QGP with u and d quarks.

References

[1] M. Ladrem, A. Ait El Djoudi, Eur. Phys. J. C **44**, 257 (2005).

[2] A. Ait El Djoudi, Canadian J. Phys. **91**, 793 (2013).

[3] L. Ghenam, A. Ait El Djoudi, K. Mezouar, Canadian J. Phys. **94**, 180 (2016).

KLEIN–GORDON OSCILLATOR USING GENERALIZED DUNKL DERIVATIVES IN THE PATH INTEGRAL FORMALISM

Djamel Eddine Mouhata **RABER**

Laboratoire LRPPS, Université de Kasdi Merbah-Ouargla
BP 511, Route Ghardaia, 30000 Ouargla, Algeria
Orcid number: 0009-0009-8489-6708
raber.djameleddinemouhata@univ-ouargla.dz

Hadjira **BENZAIR**

Laboratoire LRPPS, Université de Kasdi Merbah-Ouargla
BP 511, Route Ghardaia, 30000 Ouargla, Algeria
Orcid number: 0000-0002-2463-0520
benzair.hadjira@gmail.com

Tahar **BOUDJEDAA**

Laboratoire de Physique Théorique, Université de Jijel
BP98Ouled Aissa, 18000 Jijel, Algeria
boudjedaa@gmail.com

Mahmoud **MERAD**

Laboratoire (L.S.D.C), Université de Oum El Bouaghi
04000 Oum El Bouaghi, Algeria
0000-0001-7547-6933
meradm@gmail.com

Abstract

In this work, we study the Green's function of relativistic oscillator with spin-0 by employing the path integral formalism. Our analysis is carried out within the framework of a generalized Dunkl derivative characterized by three deformation parameters, which extends the conventional Dunkl operator. For simplicity, this generalized derivative is reformulated in terms of only two parameters, making the problem more tractable while preserving the essential features of the deformation. Within this setting, the energy spectra and the corresponding wave functions are explicitly derived as functions of the deformation parameters. Special attention is given to the role of these parameters in modifying the structure of the solutions, highlighting their influence on both the spectrum and the wave functions. Furthermore, we show that in the limiting

cases of the deformation, the system naturally reduces to the standard Dunkl derivative model. Remarkably, this recovery occurs even in the absence of Dunkl parameters, thereby confirming the consistency of our generalized framework with the conventional theory.

INSIGHTS INTO MIRROR NUCLEI STRUCTURE: A THEORETICAL SPECTROSCOPIC STUDY OF ^{21} O AND ^{21} AL

Mouna **BOUHELAL**

LPAT, Echahid Cheikh Larbi Tebessi University
12000, Tebessa, Algeria
Orcid number: 0000-0001-7346-9801
mouna.bouhelal@univ-tebessa.dz

Abir **SELIM**

LPAT, Echahid Cheikh Larbi Tebessi University
12000, Tebessa, Algeria
Orcid number: 0009-0007-0462-5496
abirselim@univ-tebessa.com

Abstract

The study of neutron-rich oxygen isotopes provides fundamental understanding of the evolution of nuclear shell structure near the drip line. In this work, we present a comprehensive shell-model investigation of the spectroscopic properties of the neutron-rich nucleus ^{2}1O, performed using our well-established PSDPF effective interaction. Excitation energies and electromagnetic transition rates have been calculated for both low- and high-lying states, enabling a detailed exploration of the underlying nuclear structure.

To extend this analysis, we also investigate the proton-rich mirror nucleus ^{2}1Al, for which electromagnetic transition strengths have been computed for all experimentally observed states. A systematic comparison between theoretical predictions and available experimental data is presented for both mirror partners.

Our results provide a robust theoretical benchmark for understanding the structure of A=21, T=5/2 nuclei and offer reliable predictions to guide future experimental investigations. This study further demonstrates the predictive power and versatility of the PSDPF interaction in describing the complex properties of neutron- and proton-rich nuclei. A detailed discussion of these findings will be presented.

THEORETICAL EXPLORATION OF THE STRUCTURE OF THE MIRROR NUCLEI ^{22} F AND ^{22} AL

Abir **SELIM**

LPAT, Echahid Cheikh Larbi Tebessi University
12000, Tebessa, Algeria
Orcid number: 0009-0007-0462-5496
abirselim@univ-tebessa.com

Mouna **BOUHELAL**

LPAT, Echahid Cheikh Larbi Tebessi University
12000, Tebessa, Algeria
Orcid number: 0000-0001-7346-9801
mouna.bouhelal@univ-tebessa.dz

Abstract

Nuclei in the sd-shell provide an ideal testing ground for nuclear structure theory, as full shell-model calculations are feasible across this mass region. However, many odd-odd nuclei remain less well thoroughly explored. Among them, the odd-odd nucleus ^{2}2F (Tz=+2) constitutes a particularly valuable case for studying shell evolution in light nuclei.

In this work, we present a theoretical shell-model analysis of ^{2}2F and its proton-rich mirror partner ^{2}2Al, performed using large-scale calculations using the $(0+1)\hbar\omega$ PSDPF effective interaction and the Nathan code. We calculated excitation energies, electromagnetic transition strengths, magnetic and quadrupole moments, and half-lives, and systematically compare them with the available experimental data.

Our results show excellent agreement with measurements, demonstrating the predictive power of the PSDPF interaction. Furthermore, the calculations provide spin–parity assignments for several states with previously uncertain or unknown configurations, offering valuable guidance for future experimental work.

INTERACTIVE SIMULATION OF ULTRASONIC CAVITATION DYNAMICS: A COMPREHENSIVE STUDY OF THE RAYLEIGH-PLESSET EQUATION

Ali Özhan **AKYÜZ**

Bucak Emin Gülmez Vocational School of Technical Sciences, Burdur Mehmet Akif Ersoy University
15300, Burdur, Türkiye
Orcid number: 0000-0001-9746-9873
aliozhanakyuz@gmail.com

Abstract

This study presents a comprehensive interactive simulation framework for modeling ultrasonic cavitation dynamics based on the Rayleigh-Plesset equation. The developed web-based simulation tool enables real-time visualization of cavitation bubble behavior under varying acoustic conditions, providing insights into the complex nonlinear dynamics of bubble oscillations. The simulation incorporates key physical parameters, including pressure amplitude, frequency, fluid density, surface tension, viscosity, and ambient pressure, allowing for systematic investigation of their effects on cavitation phenomena.

The interactive platform demonstrates the transition between stable oscillations, transient cavitation, and violent bubble collapse, offering educational and research applications in sonochemistry, biomedical ultrasound, and industrial cleaning processes. Validation against theoretical predictions shows excellent agreement for small-amplitude oscillations and captures the essential physics of large-amplitude bubble dynamics.

Keywords: Cavitation, Rayleigh-Plesset equation, Ultrasonic bubbles, Interactive simulation, Nonlinear Dynamics.

1 Introduction

Acoustic cavitation represents one of the most fascinating phenomena in fluid mechanics, where sound waves induce the formation, growth, and violent collapse of bubbles in liquids. This process has profound implications across diverse fields, from sonochemistry and biomedical applications to industrial cleaning and materials processing [1, 2].

Lord Rayleigh established the theoretical foundation for understanding cavitation dynamics, which was later refined by Plesset, leading to the renowned Rayleigh-Plesset equation. This nonlinear differential equation describes the radial motion of a spherical bubble in an infinite liquid medium under the influence of acoustic pressure variations [3, 4].

Despite its apparent simplicity, the Rayleigh-Plesset equation exhibits remarkably complex behavior, including period-doubling bifurcations, chaotic oscillations, and extreme sensitivity to initial conditions. These nonlinear dynamics make cavitation bubbles excellent model systems for studying complex physical phenomena while maintaining direct practical relevance [5, 6].

Traditional approaches to studying cavitation have relied heavily on analytical approximations or numerical solutions presented in static formats. However, the inherently dynamic nature of cavitation phenomena calls for interactive visualization tools that can provide immediate feedback on parameter variations and enable intuitive exploration of the complex parameter space [7, 8].

This work addresses this need by developing a comprehensive web-based simulation platform that combines accurate numerical modeling with real-time interactive visualization. The platform enables users to explore the rich dynamics of cavitation bubbles while maintaining computational efficiency suitable for educational and research applications.

2 Theoretical Background and Physical Mechanisms

The motion of a spherical bubble in an incompressible liquid is governed by the Rayleigh–Plesset (RP) equation, which balances inertia, the pressure difference between the gas in the bubble and the far field, surface tension, and viscous stresses. Here $R(t)$ is the instantaneous bubble radius, ρ is the liquid density, $P_g(t)$ is the gas pressure inside the bubble, $P_\infty(t)$ is the ambient (far-field) pressure, σ is the surface tension, and μ is the dynamic viscosity [5, 7, 8].

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[P_g(t) - P_\infty(t) - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R}\right]. \tag{1}$$

The cavitation process is governed by the interplay of several competing forces, starting with inertial effects, where the liquid resists rapid changes in bubble radius, leading to overshoot phenomena during both expansion and collapse phases. This inertia is constantly counteracted by surface tension, which acts to minimize the bubble surface area and provides a restoring force tending to shrink the bubble. Energy within the system is dissipated through viscous damping, which involves viscous flow around the bubble and becomes particularly significant during rapid collapse. The bubble's internal dynamics are also regulated by gas pressure, which varies according to polytropic relations dictated by heat transfer conditions. Finally, the energy input that sustains the entire process is provided by the external acoustic driving field, which powers the cavitation oscillations. The complex balance between these five mechanisms ultimately determines the bubble's fate: whether it will undergo stable oscillation, continuous growth, or violent collapse.

3 Simulation Methodology

The simulation employs a simplified yet physically meaningful approach to solve the Rayleigh–Plesset equation, utilizing a physics-based approximation to ensure real-time performance while capturing the essential dynamics. The core algorithm incorporates sinusoidal driving, modeling the acoustic pressure as

$$P(t) = P_0 + P_A \sin(\omega t),$$

and achieves multi-physics coupling by integrating the effects of density, viscosity, and surface tension through dimensionless scaling factors. To maintain numerical stability and computational efficiency, the simulation uses adaptive time stepping. Furthermore, all physical parameters can be modified via real-time parameter updates with immediate visual feedback during runtime.

The visualization system comprises a bubble animation, where the size is proportional to the instantaneous radius and color-coding indicates the oscillation phase:

- Red for large expansion ($R > 8$ μm),
- Green for strong compression ($R < 3$ μm),
- Blue for normal oscillation ($3 \leq R \leq 8$ μm).

This is complemented by dynamic graphing, which is a continuously updating plot of bubble radius versus time. It provides quantitative insight into oscillation patterns, allowing for the identification of periodic, quasi-periodic, or chaotic behavior. The interface offers seven adjustable parameters for user interaction, including: initial radius (R_0), pressure amplitude (P_A), frequency (f), liquid density (ρ), surface tension (σ), viscosity (μ), and ambient pressure (P_0).

4 Cavitation Simulation Results, Practice, and Technical Summary

The parameter sensitivity analysis of the simulation revealed that increasing pressure amplitude (PA) triggers a transition from stable oscillations to transient cavitation, with violent collapse events and explosive rebound becoming frequent at high amplitudes. The frequency dependence exhibited resonance-like behavior tied to initial size and fluid properties, though higher frequencies generally reduced oscillation amplitudes due to inertial limitations. Regarding fluid property influences, higher density increases inertial overshoot. Increased viscosity provides damping that stabilizes oscillations but reduces maximum amplitudes, while surface tension acts as a restoring force promoting faster collapse. The analysis identifies three distinct dynamic regimes: Stable Oscillations (periodic, low amplitude), Transient Cavitation (large, highly nonlinear oscillations), and Violent Collapse (generating intense local pressures). For educational applications, the interactive platform enhances conceptual understanding by visualizing abstract concepts like nonlinear dynamics, enabling systematic parameter exploration of cause-and-effect, and supporting quantitative analysis through dynamic graphing. Technically, the simulation is a self-contained, pure client-side HTML5/JavaScript application. It ensures maximum accessibility and broad compatibility with responsive design by leveraging features like real-time parameter adjustment and optimized mathematical operations for performance. The platform has been validated by showing excellent agreement with analytical solutions in the linear regime and qualitative matching with full Rayleigh-Plesset solutions for large-amplitude behavior. Despite its limitations, which include the exclusion of some nonlinear effects, compressibility, and heat transfer, the simplified model successfully captures the essential physics of cavitation phenomena.

Figure 1 shows a screenshot of the simulation.

Figure 1: Simulation interface and real-time bubble dynamics visualization.

Conclusion

This work successfully presents a comprehensive interactive simulation platform for exploring ultrasonic cavitation dynamics based on the Rayleigh-Plesset equation. The platform's validation showed excellent agreement with analytical solutions for small-amplitude oscillations, and its qualitative behavior for large-amplitude oscillations matches both experimental observations and full numerical solutions. Despite this success, limitations exist due to the simplified mathematical model, which does not explicitly include all nonlinear effects, liquid compressibility, vapor pressure, heat transfer, or gas diffusion, and the assumption of spherical symmetry may break down during violent collapse. Nevertheless, the simulation effectively captures the essential physics of cavitation phenomena. The platform's key contributions include its real-time interactive visualization, its utility as an educational tool for teaching complex nonlinear concepts through hands-on experimentation, its role as a foundation for research, and its technical innovation as a purely web-based implementation that maximizes accessibility. The simulation successfully demonstrates the rich dynamics of cavitation bubbles, from stable oscillations to violent collapse events, marking a significant advance in making cavitation physics accessible to diverse students and researchers. Future developments are planned to incorporate additional physical effects and extend its use to specialized applications like sonochemistry.

References

[1] K. Yasui, *Acoustic cavitation*. In *Acoustic Cavitation and Bubble Dynamics*, pp. 1–35. Cham: Springer International Publishing, 2017.

[2] J. Li, D. Li, and T. Li, *Progress in the Understanding of Cavitation and Its Related Applications. Fluid Dynamics & Materials Processing* **21**(3), 2025.

[3] R. Roohi, S. M. B. Hashemi, D. Jafarpour, and M. Akbari, *Modeling and Simulation of Cavitation in Sono-Processes*. In *Modeling and Simulation of Sono-Processes*, pp. 123–137. Elsevier, 2025.

[4] L. Toledo, K. Choi, H. Kim, and C. Kim, *Generalized modeling of cavitation dynamics encompassing multiple cavitation regimes*. SSRN 4972155, 2025.

[5] J. P. Franc, *The Rayleigh–Plesset equation: a simple and powerful tool*. In *Cavitation and Bubble Dynamics in Turbomachinery and Cavitating Turbopumps*, pp. 1–41. Vienna: Springer, 2007.

[6] D. Fuster, *A review of models for bubble clusters in compressible cavitating flows. Flow, Turbulence and Combustion* **102**(3), 497–536, 2019.

[7] B. Jia and H. Soyama, *Non-spherical cavitation bubbles: A review. Fluids* **9**(11), 249, 2024.

[8] C. E. Brennen, *Cavitation and Bubble Dynamics*. Cambridge University Press, 2014.

ENHANCED MUTUAL INDUCTANCE CALCULATION FOR LATERALLY MISALIGNED CIRCULAR COILS IN WIRELESS POWER TRANSFER SYSTEMS

Muzaffer, **ERDOĞAN**

Physics Department, Faculty of Arts and Sciences, Tekirdağ Namik Kemal University
Namık Kemal Mah. Kampüs Cad. No:1 59030 Süleymanpaşa - Tekirdağ
Orcid number: 0000-0001-8738-2299
merdogan@nku.edu.tr

Abstract

This talk introduces an enhanced computational method for accurately calculating mutual inductance between misaligned circular coils in wireless power transfer (WPT) systems. The method determines the radial and axial magnetic field components of the primary coil and integrates their normal component over the secondary coil plane to obtain the magnetic flux. Mutual inductance is then derived by dividing this flux by the excitation current. Validation against theoretical and experimental results for lateral displacement scenarios shows excellent agreement with existing literature, confirming the method's accuracy and reliability. The approach combines computational efficiency with improved predictive capability, making it a valuable tool for the design and optimization of WPT systems.

1 Introduction

Wireless power transfer technology has gained significant momentum in recent years, particularly for electric vehicle charging, consumer electronics, and biomedical applications [1,2]. The electromagnetic coupling between transmitter and receiver coils determines the power transfer efficiency, making accurate mutual inductance prediction essential for system optimization [3,4].

In real-world applications, perfect coil alignment is rarely achieved due to manufacturing tolerances, installation constraints, and dynamic positioning requirements [5]. Lateral misalignment, where coils are displaced horizontally while maintaining parallel planes, represents one of the most common practical scenarios in wireless power transfer systems [6,7]. This displacement significantly affects mutual inductance and consequently the overall system performance.

Traditional mutual inductance calculations rely on Neumann's formula, which involves complex double line integrals over current paths [8]. For laterally misaligned coils, these calculations become mathematically intensive and computationally demanding [9]. Various analytical approximations have been developed, including series expansions and elliptic integral representations, but these often involve trade-offs between accuracy and computational efficiency [10,11].

Recent research has explored alternative computational strategies to address these limitations. Finite element methods provide high accuracy but are computationally expensive for design optimization tasks [12]. Semi-analytical approaches attempt to balance accuracy with computational speed but may introduce approximation errors for significant misalignments [13].

The magnetic field integration approach offers a different perspective on mutual inductance calculation. By directly computing the magnetic field components generated by the primary coil and integrating the flux through the secondary coil area, this method can potentially avoid some of the mathematical complexities inherent in Neumann-based formulations [14]. This approach is particularly well-suited for lateral misalignment scenarios where the geometric relationships can be expressed more naturally in terms of field distributions.

This paper presents the application of magnetic field component integration specifically for lateral misalignment cases. The method calculates radial and axial magnetic field components and determines mutual inductance through flux integration. Validation against experimental data from literature demonstrates the effectiveness of this approach for practical wireless power transfer system design [15].

2 Theoretical Background

In this work, we calculate the mutual inductance between two parallel and laterally misaligned circular coils. Figure 1 illustrates the configuration of the two coils. The primary coil lies in the $x-y$ plane, centered at the origin, while the secondary coil is in the $z=p$ plane, with its center located at the point $(0, c, p)$.

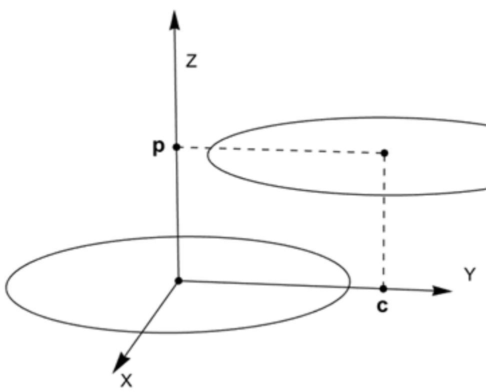

Figure 1: Laterally displaced parallel plane coils

The radial and axial magnetic field components, B_r and B_z, generated by an electric current circulating counterclockwise in the primary coil are depicted in the figure.

To calculate the magnetic flux, we first determine the off-axis radial and axial magnetic field components produced by the current in the primary coil. These components are given by the equations 1 and 2 [16, 17]:

$$B_r(z,r) = \frac{\mu_0 \, I \, z}{2\pi \, r \, \sqrt{(a+r)^2+z^2}} \left(E(k) \frac{a^2+r^2+z^2}{(a-r)^2+z^2} - K(k) \right) \tag{1}$$

$$B_z(z,r) = \frac{\mu_0 \, I}{2\pi \, \sqrt{(a+r)^2+z^2}} \left(E(k) \frac{a^2-r^2-z^2}{(a-r)^2+z^2} + K(k) \right) \tag{2}$$

where $E(k)$ and $K(k)$ are the complete elliptic integrals of the first and second kinds, respectively. Here, a is the common radius of the coils, r is the radial distance, and z is the vertical distance. The argument of the complete integrals is given by:

$$k = \frac{4a\,r}{(a+r)^2 + z^2}. \tag{3}$$

The magnetic flux Φ through the secondary coil, generated by the current I in the primary coil, is given by the surface integral of the normal component of the magnetic field over its surface area.

$$\Phi = \int \vec{B} \cdot d\vec{A} = \int B(r) dA \tag{4}$$

where $B(r)$ is the sum of the normal components of the radial and axial magnetic fields, given in Equations 1 and 2.

$$B(r) = (B_r\,\hat{r} + B_z\,\hat{z}) \cdot \hat{n} \tag{5}$$

where $\hat{n}$ is the unit normal vector of the secondary coil plane. When the secondary coil is rotated about the x-axis by an angle θ, this unit vector becomes $\hat{n} = \sin\theta\,\hat{y} + \cos\theta\,\hat{z}$. In the special case where $\theta = 0$, the vector reduces to $\hat{n} = \hat{z}$, and the magnetic field component simplifies to $B(r) = B_z$. For a fixed value of z, we omit the z dependence from the argument. Therefore,

$$B(r) = \frac{\mu_0\,I}{2\pi\,\sqrt{(a+r)^2 + z^2}} \left(E(k)\frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} + K(k) \right) \tag{6}$$

By substituting the expression for the normal magnetic field from Equation 6 into the flux calculation in Equation 4, we obtain the integral with explicit limits for the secondary coil, whose center is located at the point $(0, 0, z)$, assuming no lateral or angular misalignment:

$$\Phi = \int_0^{2\pi} \int_0^{a-d/2} B(r)\,r\,dr\,d\phi, \tag{7}$$

where a is the common radius and d is the common diameter of the circular coils. To account for a lateral misalignment by a distance c, we replace r with $\sqrt{r^2 + c^2 + 2rc\sin\phi}$. Additionally, exploiting the symmetry of the system about the $y - z$ plane, we modify the limits of the ϕ integral from $-\pi/2$ to $\pi/2$, and multiply the integral by 2.

$$\Phi = 2\int_{-\pi/2}^{\pi/2} \int_0^{a-d/2} B(\sqrt{r^2 + c^2 + 2rc\sin\phi})\,r\,dr\,d\phi, \tag{8}$$

In the case where both coils are single-turn, the mutual inductance is given by Equation 9.

$$m = \frac{\Phi}{I}, \tag{9}$$

which is independent of the current and depends on the geometric parameters and μ_0. In the case where the coils are multi-turn—for instance, with N_1 and N_2 turns for the primary and secondary coils, respectively—the mutual inductance is given by Equation 10.

$$m = N_1\,N_2\,\frac{\Phi}{I}, \tag{10}$$

assuming that the turns do not significantly alter the radius or the length of the coils.

3 Results and Discussion

Calculations are made for the numerical values of the parameters $z = 10\,cm$, $a = 10\,cm$, $d = 2\,mm$, $N_1 = 5$, $N_2 = 5$ turns, the variation of the mutual inductance m versus lateral misalignment c is given in Figure 2

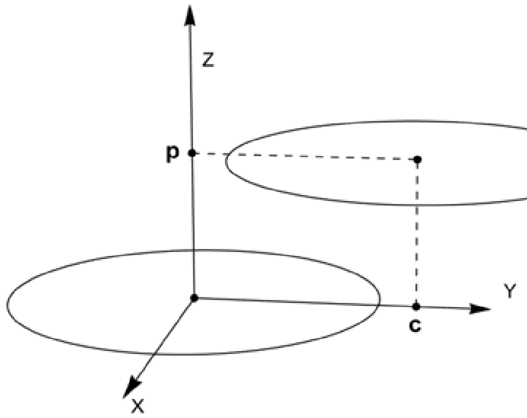

Figure 2: Calculated mutual inductance versus lateral distance.

After a certain lateral distance, the mutual inductance m drops to slightly negative values, as indicated in Figure 2. This phenomenon is attributed to the reversal of the axial magnetic field component produced by the primary coil current at that point. These results are in excellent agreement with those reported in [15] and are also consistent with their experimental findings.

4 Conclusion

The magnetic field integration method provides an effective alternative for calculating mutual inductance in laterally misaligned circular coil systems. This approach shows superior agreement with experimental measurements compared to traditional analytical methods, especially for significant lateral displacements. It offers practical advantages for wireless power transfer system design, where both computational efficiency and accuracy are critical. Our ongoing work involves analyzing the variation of mutual inductance with respect to axial displacement, rotational misalignment, as well as combined rotational and lateral misalignment. These calculations aim to encompass all possible relative orientations. The detailed results will be presented in our future publications.

References

[1] A. Kurs et al., Science **317**, 83 (2007).

[2] S. Li and C. C. Mi, IEEE J. Emerging Sel. Topics Power Electron. **3**, 4 (2015).

[3] X. Lu et al., Proc. IEEE **103**, 1271 (2015).

[4] W. Zhang and C. C. Mi, IEEE Trans. Veh. Technol. **65**, 4768 (2016).

[5] K. Fotopoulou and B. W. Flynn, IEEE Trans. Magn. **47**, 416 (2011).

[6] W. X. Zhong et al., IEEE Trans. Power Electron. **29**, 4500 (2014).

[7] S. Chopra and P. Bauer, IEEE Trans. Ind. Electron. **60**, 3506 (2013).

[8] F. W. Grover, Inductance Calculations, (Dover Publications, New York, 1946).

[9] S. I. Babic and C. Akyel, IEEE Trans. Magn. **44**, 1743 (2008).

[10] C. Akyel et al., Prog. Electromagn. Res. **91**, 287 (2009).

[11] S. Babic and C. Akyel, IEEE Trans. Magn. **36**, 1970 (2000).

[12] J. Acero et al., IEEE Trans. Ind. Electron. **60**, 410 (2013).

[13] S. R. Khan et al., IEEE Trans. Microw. Theory Techn. **66**, 4158 (2018).

[14] N. Pirincci and H. Altun, IEEE Trans. Magn. **58**, 1 (2022).

[15] X. Zhang et al., J. Eng. **2019**, 1041 (2019).

[16] Tables of Integrals and Other Mathematical Data, H. B. Dwight, (The Macmillan Company, New York, 1961).

[17] H. B. Dwight, Electrical Engineering **54**, 709 (1935).

EXACT NONLINEAR SOLUTIONS OF THE SASA–SATSUMA EQUATION

Zeyneb **TAIBI**

Laboratory for Theoretical Physics and Material Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
Oulad Fares, Chlef, 02000, Algeria
Orcid number: 0009-0009-4371-1439
Email: z.taibi@univ-chlef.dz

Houria **CHAACHOUA SAMEUT**

Laboratory for Theoretical Physics and Material Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
Oulad Fares, Chlef, 02000, Algeria
Orcid number: 0009-0000-4387-4048
chachou_houria@yahoo.fr

Fateh **MERABTINE**

Laboratory for Theoretical Physics and Material Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
Oulad Fares, Chlef, 02000, Algeria
Orcid number: 0009-0008-9850-4800
Email: f.merabtine@univ-chlef.dz

Abstract

We are now in a position to interpret a wide range of physical phenomena—both real and hypothetical—as approximations arising from soliton theory. By focusing on soliton solutions, we extend these methods to nonlinear equations of the nonlinear Schrödinger (NLS) type. Solitons play a crucial role in diverse fields such as hydrodynamics, quantum gases, optical fiber communications, and plasma physics, owing to their stable, localized wave patterns. To derive exact solutions, we employ the Darboux transformation together with a Lax pair, ensuring the integrability condition is satisfied. Starting from a simple seed solution, we recover soliton-like structures analytically. By modifying the seed, we obtain qualitatively different solutions, namely breather modes.

ANALYTICAL STUDY OF QUANTUM DEFORMATION EFFECTS ON CaF AND BaH MOLECULES IN (ANTI)-DE SITTER SPACES WITH APPLICATION TO QUANTUM COMPUTING

Meriem **ABDELAZIZ**
LPPNM Laboratory, Department of Matter Sciences, University of Biskra
07000 Biskra, Algeria
Orcid number: 0000-0002-8191-9697
meriem.abdelaziz@univ-biskra.dz

Mustapha **MOUMNI**
LPRIM Laboratory, Department of Physics, University of Batna I
05000 Batna, Algeria
Orcid number: 0000-0002-8096-6280
mustapha.moumni@univ-batna.dz

Mokhtar **FALEK**
Department of Physics, University of Khenchela, 40000 Khenchela, Algeria
Orcid number: 0000-0002-0466-9559
mokhtar.falek@univ-khenchela.dz

Abstract

This work presents an analytical study of the Schrödinger equation in de Sitter (dS) and anti-de Sitter (AdS) spaces for the pseudoharmonic oscillator (PHO) potential. Using the extended uncertainty principle (EUP) and the Nikiforov–Uvarov method, we derive exact energy eigenvalues and eigenfunctions for diatomic molecules.

The study is focused on CaF and BaH, two polar molecules of particular interest for quantum computing because of their strong dipole moments, controllable rovibrational states, and long coherence times. Our findings reveal that quantum deformation in curved space-times produces non-uniform shifts in the bound-state spectrum, with possible level inversions at critical deformation parameters. These modifications may alter ground state definitions and influence the robustness of molecular qubits.

The results provide both fundamental insights into deformed quantum mechanics and practical implications for quantum technologies based on cold diatomic molecules.

References

[1] M. Abdelaziz, M. Moumni, M. Falek, Int. J. Theor. Phys. **63**, 237 (2024).

[2] M. I. Samar, V. M. Tkachuk, Phys. Lett. A **530**, 130146 (2025).

[3] W. S. Chung, A. Algin, Phys. Scr. **99**, 055229 (2024).

[4] I. G. Avramidi, R. Niardi, (2024). *Geometric Deformation of Quantum Mechanics*, arXiv:2412.08007 [quant-ph].

DISORDER-INDUCED LOCALIZATION OF EXPANDING QUANTUM DROPLETS

Zohra **MEHRI** [1]

Department of Physics, Faculty of Sciences and Technology, Ahmed Zabana University of Relizane
Bourmadia, BP 48000, Relizane, Algeria
Orcid number: 0009-0006-3643-9623
zohra.mehri@univ-relizane.dz

Abdelaali **BOUDJEMAA**

Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
P.O. Box 78, 02000, Ouled-Fares, Chlef, Algeria.
a.boudjemaa@univ-chlef.dz

Abstract

We study the expansion of a one-dimensional quantum droplet in a speckle potential using the generalized Gross-Pitaevskii equation. We compute the droplet width, density profiles, diffusion exponent and coefficient, and localization length for both small and large droplets. Interesting classes of anomalous diffusion are observed in the transport dynamics, ranging from hyperballistic behavior in the free case to subdiffusion under strong disorder. We find that above a certain critical disorder strength, the droplet undergoes a transition to Anderson localization.

References

[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

[2] G. Roati, C. D'Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, M. Inguscio, Nature **453**, 895 (2008).

[3] Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer, A. Aspect, Nature **453**, 891 (2008).

[1] This contribution was not presented at the conference.

A THREE-PARTY LIGHTWEIGHT QUANTUM KEY DISTRIBUTION PROTOCOL IN A RESTRICTED QUANTUM ENVIRONMENT

Mustapha Anis **YOUNES**

Université de Bejaia, Faculté des Sciences Exactes, Laboratoire de Physique Théorique,
06000 Bejaia, Algérie
Orcid number: 0009-0007-4979-9887
mustaphaanis.younes@univ-bejaia.dz

Sofia **ZEBBOUDJ**

ENSIBS, Université Bretagne Sud,
56000, Vannes, France
Orcid number: 0000-0002-5721-5753
sofia.zebboudj@univ-ubs.fr

Abdelhakim **GHARBI**

Université de Bejaia, Faculté des Sciences Exactes, Laboratoire de Physique Théorique,
06000 Bejaia, Algérie
Orcid number: 0000-0003-2995-3238
abdelhakim.gharbi@univ-bejaia.dz

Abstract

Quantum key distribution (QKD) protocols enable two parties to establish a shared secret key that can later be used to encrypt and decrypt confidential information. By leveraging the fundamental laws of quantum physics, such keys remain secure against an all-powerful quantum adversary. However, most existing protocols require all participants to possess full quantum capabilities, which is unrealistic since not all users can afford or operate advanced quantum devices. Moreover, extending QKD to multipartite scenarios is an active area of research due to its applications, such as broadcasting.

In this work, we propose a novel three-party lightweight quantum key distribution (LQKD) protocol based on the four-particle cluster state within a quantum-restricted environment. The protocol enables a quantum-capable user to simultaneously establish two separate secret keys with two "classical" users who hold limited quantum capabilities. By employing a one-way qubit transmission method, our scheme addresses several limitations found in similar schemes: (1) it eliminates the need for classical participants to use costly quantum devices to defend against quantum Trojan horse attacks; (2) it shortens the qubit transmission distance; and (3) it achieves higher qubit

efficiency. Consequently, the proposed LQKD protocol is both more lightweight and more practical than existing schemes. Furthermore, it is proven unconditionally secure, with a noise tolerance close to that of BB84.

THERMAL PROPERTIES OF THE DIRAC OSCILLATOR IN NONCOMMUTATIVE PHASE SPACE

Lakhdar **SEK**

NTDL Laboratory, Faculty of Technology, University of El Oued, 39000 El Oued, Algeria
Orcid number: 0009-0004-0582-5949
sek-lakhdar@univ-eloued.dz

Abstract

This study investigates the Dirac oscillator within a deformed noncommutative phase space framework, emphasizing its thermal properties. By employing the canonical ensemble approach and utilizing the Euler–Maclaurin formula, we derive the partition function and compute key thermodynamic quantities, including Helmholtz free energy, entropy, mean energy, and heat capacity. Our analysis reveals that the noncommutative deformation introduces significant modifications to these properties compared to their commutative counterparts. Notably, we observe the emergence of re-entrant phase transitions in the heat capacity as a function of temperature, indicating complex thermal behaviors influenced by the underlying noncommutative geometry. These findings provide deeper insights into the interplay between quantum mechanics, spacetime structure, and thermal dynamics, with potential implications for high-energy physics and cosmology.

References

[1] R. R. S. Oliveira and R. R. Landim, Acta Phys. Pol. B **55**, 10-A1 (2022).

[2] A. Boumali, Eur. Phys. J. Plus **128**, 124 (2013).

[3] A. M. Frassino, arXiv:1707.06984 (2017).

[4] A. Maireche, Rev. Mex. Fis. **71**, 020401 (2025).

[5] I. Haouam, Sci. Rep. **15**, 28771 (2025).

EFFECT OF NON–COMMUTATIVE GEOMETRY ON OPTICAL APPEARANCE OF A REGULAR BLACK HOLE

Narges **HEIDARI**

Center for Theoretical Physics, Khazar University
41 Mehseti Street, Baku, AZ-1096, Azerbaijan.
0000-0002-4623-8909
heidari.n@gmail.com

Adailton **A.A.A.Filho**

Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051–970, João Pessoa, Paraíba, Brazil.
dilto@fisica.ufc.br

Iarley **P. LOBO**

Department of Chemistry and Physics, Federal University of Paraíba,
Rodovia BR 079 - km 12, 58397-000 Areia-PB, Brazil.
lobofisica@gmail.com

Abstract

The reconciliation of general relativity with quantum gravity remains a central challenge in theoretical physics. Non-commutative geometry offers a promising path toward this goal by introducing a fundamental minimal length. In this work, we present a novel regular black hole solution derived from a non-commutative gauge theory incorporating a $\partial_r \wedge \partial_\theta$ Moyal twist. We analyze the fundamental properties of this Hayward-like metric, confirming its regularity and exploring its horizon structure. Furthermore, we investigate the black hole's dynamical features by computing its quasinormal modes for scalar perturbations. The characteristics of the photon sphere and shadow are studied through null geodesics and the Gaussian curvature, while the topological feature is analyzed. Finally, we derive stringent constraints on the non-commutativity parameter Θ and the Hayward parameter l using EHT observational data. This work provides a comprehensive framework for testing non-commutative geometry through astrophysical observation.

FUNDAMENTAL FREQUENCIES AND THERMODYNAMICAL PROPERTIES OF BLACK HOLES IN THE PRESENCE OF EXOTIC MATTER

Bekzod **RAHMATOV**

Samarkand State University
University blv.15, Samarkand 140104, Uzbekistan
Orcid number: 0009-0001-0394-650X
rahmatovbekzod@samdu.uz

Abstract

This work investigates the fundamental frequencies and thermodynamical properties of black holes surrounded by exotic matter fields within the framework of general relativity and nonlinear electrodynamics (NED). The study addresses three major scenarios: (i) black holes embedded in Chaplygin-like dark matter, (ii) black holes coupled with NED and surrounded by perfect fluid dark matter (PFDM), and (iii) magnetically charged black holes in PFDM backgrounds.

For the Chaplygin-like dark matter case, we analyze the spacetime geometry, horizon structure, and quasi-periodic oscillations (QPOs), showing the emergence of both ISCO and OSCO orbits and their dependence on exotic matter parameters. In the NED–PFDM framework, we obtain new black hole solutions and study their thermodynamic properties, including Hawking temperature, entropy, and specific heat, revealing stability regions and phase transition behavior. In the case of magnetically charged black holes with PFDM, we explore particle dynamics, radiation processes, and QPOs, applying Markov Chain Monte Carlo (MCMC) methods to constrain black hole and exotic matter parameters using observational QPO data from microquasars and supermassive black holes.

The results demonstrate that exotic matter fields and nonlinear electrodynamics significantly modify black hole geometry, stability, and thermodynamics, providing new insights into observable astrophysical phenomena. These findings contribute to bridging theoretical predictions with observational data from X-ray binaries and galactic centers, offering potential tests for extended gravity and dark matter models.

MICROSCOPIC ANALYSIS OF THE MEAN SQUARE CHARGE RADIUS $\langle R^2 \rangle$ IN EVEN–EVEN CERIUM (Ce) ISOTOPES, BASED ON DEFORMATION ENERGIES AND DEFORMATION PARAMETERS

Hadj **MOULOUDJ**[1]

Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba
B.P. 92, Vieux-Kouba 16050, Algiers, Algeria
Orcid number: 0009-0006-0376-9642
hadj.mouloudj@gmail.com

Oussama **ZEGGAI**

Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Ben Bouali University BP 151, Chlef 02180, Algeria

Belarbi **MOUSAAB**

Laboratory of Micro and Nanophysics — LaMiN, Department of FPST-Ecole Nationale Polytechnique d'Oran-Maurice Audin
BP 1523, Oran 31000, Algeria

Fethi **REDJEM**

Department of Material Sciences, Faculty of Science, Amar Telidji University.
35 G Gherdaia road -03000, Laghouat, Algeria

Abstract

This study investigates the mean square charge radius [1] and deformation energies of even–even nuclei within the mass range $116 \leq A \leq 140$, focusing on cerium isotopes. A microscopic approach is employed, combining the liquid drop model for macroscopic energy with Strutinsky shell corrections and BCS (Bardeen–Cooper–Schrieffer) pairing correlations for microscopic refinements. The shell corrections are derived by solving a Schrödinger equation with a deformed Woods–Saxon mean field. Numerical solutions, performed using an optimized FORTRAN program, identify equilibrium shapes by locating energy minima in deformation energy contour plots. From these equilibrium shapes, single–particle wave functions and the BCS approximation are used to calculate the mean square charge radius [2]. Theoretical results align closely with experimental data, confirming the robustness of the proposed model. This study ad-

[1]This contribution was not presented at the conference.

vances our understanding of nuclear deformation and structure, particularly in isotopes far from spherical symmetry.

References

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol. I & II* (World Scientific, 1998).

[2] P.-G. Reinhard, W. Nazarewicz, Phys. Rev. C **81**, 051303 (2010).

$Cs_2AlCuCl_6$: A NEW CLASS OF LEAD-FREE DOUBLE PEROVSKITES WITH PROMISING STRUCTURAL, MECHANICAL, ELECTRONIC AND OPTICAL PROPERTIES

Amel **MERRAD**

Laboratory for Theoretical Physics and Material Physics, Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
P.O. Box 78, 02000, Ouled Fares, Algeria
Orcid number: 0009-0004-1264-5348
a.merrad@univ-chlef.dz

Halima **BOUCHENAFA**

Laboratory for Theoretical Physics and Material Physics, Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
P.O. Box 78, 02000, Ouled Fares, Algeria
Orcid number: 0000-0002-7674-9865
bouchenafa_halima@yahoo.fr

Boucif **BENICHOU**

Department of Electronics, Faculty of Technology,Hassiba Benbouali University of Chlef
P.O. Box 78, 02000, Ouled Fares, Algeria
Orcid number: 0000-0002-2049-3813
boucif_benichou@yahoo.fr

Youssef **SEKSAK**

Physics Department, Physico-Chemistry of Materials and Environment, Ziane Achour University of Djelfa
Djelfa, BP 3117, Algeria

Samia **RADJEL**

Signals Systems Artificial Intelligence Laboratory (2SAIL), Faculty of Technology, Hassiba Benbouali University of Chlef
Chlef, 02000, Algeria

Abstract

This work presents a comprehensive first-principles study of the structural, mechanical, electronic and optical properties of the novel lead-free double perovskite semiconductor $Cs_2AlCuCl_6$. The full-potential linearised augmented plane wave ($FP-LAPW$) method was employed within the framework of density functional theory

(DFT). We analysed the energetic stability, Goldschmidt tolerance factor (τ) and mechanical properties of this compound. Our findings confirm that $Cs_2AlCuCl_6$ has a perfectly cubic structure within the $Fm-3m$ space group. Furthermore, the mechanical analysis revealed that this material is highly elastic and strong, with high bulk and Young's moduli, highlighting its mechanical robustness. Electronic structure calculations demonstrate that $Cs_2AlCuCl_6$ has direct band gaps of 0.63 eV (GGA) and 1.81 eV (mBJ). Additionally, this compound exhibits significant optical absorption in the visible spectrum, suggesting its potential application in energy conversion technologies. To our knowledge, this is the first study to provide a predictive analysis of $Cs_2AlCuCl_6$, as no previous theoretical or experimental data is available for comparison.

TWO-COMPONENT BOSE GASES AT ULTRA-COLD TEMPERATURE

Atika **MEHEDI**[1]

Laboratory for Theoretical Physics and Material Physics, Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef
P.O. Box 78, 02000, Ouled Fares, Algeria
Orcid number: 0000-0003-4961-9536
a.mehedi@univ-chlef.dz

Abstract

Binary mixtures and especially mixtures of cold molecules is by now a challenging field of research both theoretical, numerical and experimental. In this preliminary work, we construct a variational approximation scheme to derive the mean- field dynamical equations for mixtures of atoms and molecules at low temperature. The anomalous averages and the non condensate densities for a 1D homogeneous bose mixture at zero temperature are computed numerically.

[1]This contribution was not presented at the conference.